Univerzitet u Sarajevu
Elektrotehnički fakultet Sarajevo

Rješenja zadataka sa kvalifikacionog ispita za prvi ciklus studija u akademskoj 2009/2010. godini

- Grupa B -

1. zadatak:

$$y=x^2+x-6$$

1. nule:

Diskriminanta je $D=b^2-4ac=1^2-4\cdot 1\cdot(-6)=25$

$$x_{1/2}=\frac{-1\pm\sqrt{25}}{2}=\frac{-1\pm 5}{2}$$

$$x_1=-3, \quad x_2=2.$$

2. znak:

Kako je a=1>0 slijedi y>0 za $x\in(-\infty, x_1)\cup(x_{2,}+\infty)$

i y<0 za $x\in(x_{1,} x_2)$

tako je za $x\in(-\infty,-3)\cup(x,+\infty)$ $y>0$

$x\in(-3,2)$ $y<0$

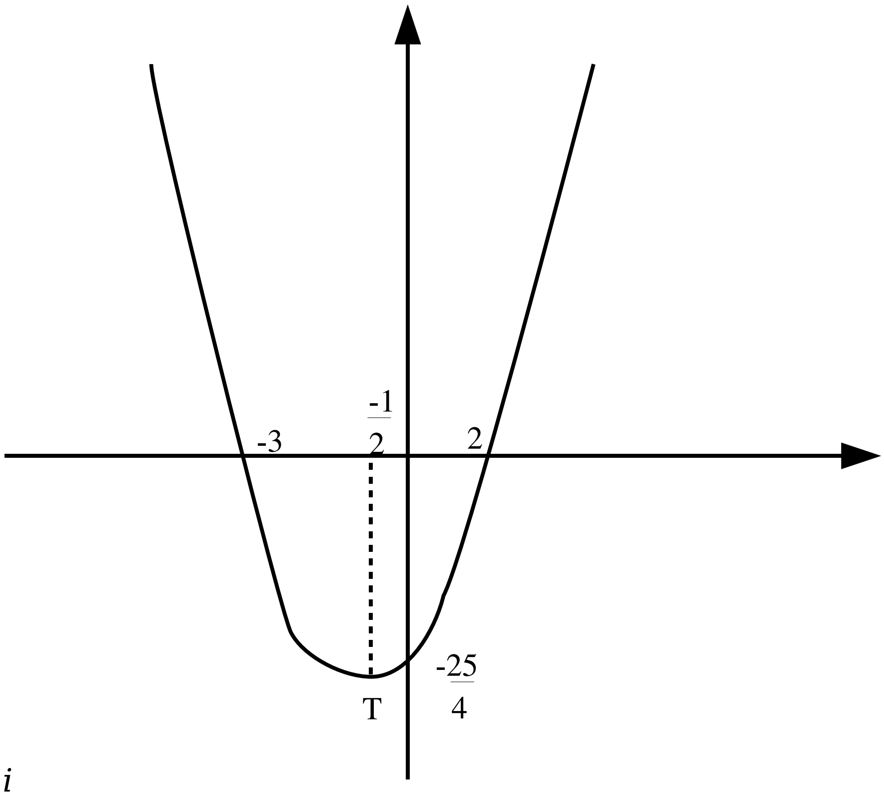
3. tjeme:

$$x_T=-\frac{b}{2a}=-\frac{1}{2}$$

$$y_T=y(x_T)=(-\frac{1}{2})^2+(-\frac{1}{2})-6=\frac{1}{4}-\frac{1}{2}-6=-\frac{25}{4}$$

TJEME: $(x_T, y_T)=(-\frac{1}{2},-\frac{25}{4})$

4. grafik:



2. zadatak:

$$z=\frac{1-3i}{1-i}-\frac{i}{2+i}$$

$$z=\frac{(1-3i)(2+i)-i(1-i)}{(1-i)(2+i)}=\frac{4-6i}{3-i}$$

$$z=\frac{4-6i}{3-i}\cdot\frac{3+i}{3+i}=\frac{18-14i}{10}=\frac{9-7i}{5}$$

Opšti oblik trigonometrijskog i eksponencijalnog prikaza:

$$|z|=\rho=\sqrt{\left(\frac{9}{5}\right)^2+\left(-\frac{7}{5}\right)^2}=\frac{\sqrt{130}}{5}$$

$$\varphi=2\pi-arctg\left(\frac{\frac{7}{5}}{\frac{9}{5}}\right)=2\pi-arctg\left(\frac{7}{9}\right)$$

Kvadrant gdje je z

$$Z=\rho\cdot(\cos\varphi+i\cdot\sin\varphi)=\frac{\sqrt{130}}{5}\cdot\left(\cos\left(arctg\left(\frac{7}{9}\right)\right)-i\cdot\sin\left(arctg\left(\frac{7}{9}\right)\right)\right)$$

$$Z=\rho\cdot e^{i\cdot\varphi}=\frac{\sqrt{130}}{5}\cdot e^{i(2\pi-arctg(\frac{7}{9}))}=\frac{\sqrt{130}}{5}\cdot e^{-i\cdot arctg(\frac{7}{9})}$$

3. zadatak:

$$\sqrt{\frac{x^2-3x+2}{x-3}}<2$$

Definiciono područje: rastavimo x^2-3x+2 kao $(x-1)(x-2)$ pa je D.P. dato za $x\neq3$ i

$$\frac{x^2-3x+2}{x-3} \geqslant 0$$

$$\frac{x^2-3x+2}{x-3} = \frac{(x-1)(x-2)}{x-3} \geqslant 0$$

	- ∞ … 1	1 … 2	2 … 3	3 … + ∞
x-1	-	+	+	+
x-2	-	-	+	+
x-3	-	-	-	+
$\frac{(x-1)(x-3)}{x-2}$	-	+	-	+

Definiciono područje:

$x \in [1,2] \cup (3,+\infty)$

$$\sqrt{\frac{x^2-3x+2}{x-3}} < 2 \quad / \;^2$$

$$\frac{x^2-3x+2}{x-3} < 4 \qquad\qquad \frac{x^2-3x+2-4x+12}{x-3} < 0$$

(*) $$\frac{x^2-7x+14}{x-3} < 0$$

$x^2-7x+14$ ima diskriminantu $D=(-٧)^{٢}-٤\cdot١\cdot١٤=٤٩-٥٦<٠$ pa je

(**) $x^2-7x+14>0 \quad \forall x \in \mathbb{R}$

Da bi (*) bila ispunjena, uz činjenicu (**) slijedi x=3<0 ==> x<3 tj. $x \in (-\infty,3)$

U presjeku sa DP daje konačno rješenje:

$x \in [1,2] \cup (3,+\infty) \cap (-\infty,3)$ tj. $x \in [1,2]$

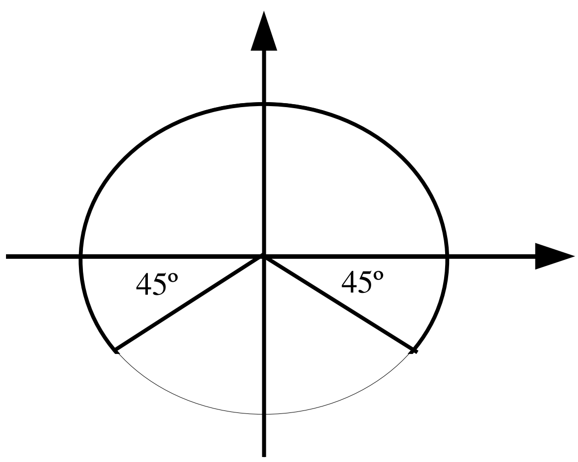
4. zadatak:

$$\sqrt{3}\cdot\cos 4x+\sin 4x > -\sqrt{2} \quad / :2$$

$$\frac{\sqrt{3}}{2}\cdot\cos 4x+\frac{1}{2}\sin 4x > -\frac{\sqrt{2}}{2}$$

$$\sin\frac{\pi}{3}\cos 4x+\cos\frac{\pi}{3}\sin 4x > -\frac{\sqrt{2}}{2}$$

(1) $$\sin(4x+\frac{\pi}{3}) > -\frac{\sqrt{2}}{2}$$



Rješenje (1) je dato na slici iznad sa:

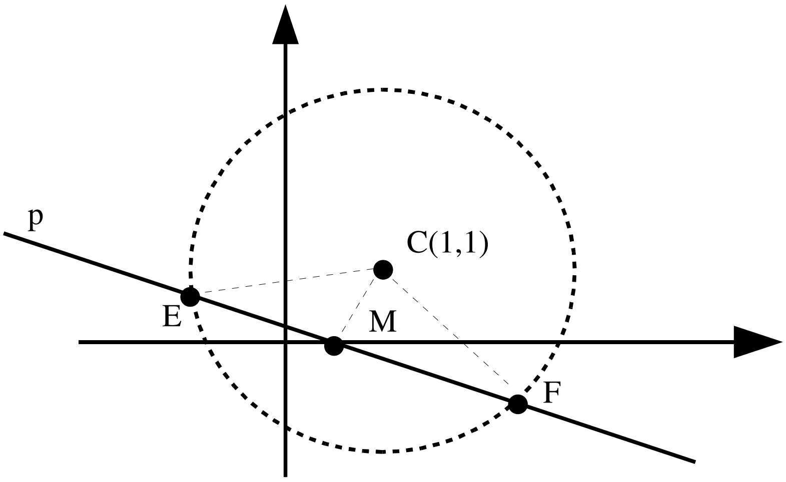
$$(4x+\frac{\pi}{3})\in\ [\ 2k\pi,\frac{5\pi}{4}+2k\pi\ )\cup\ (\frac{7\pi}{4}+2k\pi,2\pi+2k\pi)$$

$k=0,\pm1,\pm2,...$

odnosno:

$$x\in\ [\ -\frac{\pi}{12}+\frac{k\pi}{2};\frac{11\pi}{48}+\frac{k\pi}{2}\ )\cup(\ \frac{17\pi}{48}+\frac{k\pi}{2};\frac{5\pi}{12}+\frac{k\pi}{2}\ )$$

5. zadatak:



$$\overline{EF}=\frac{16}{5}$$

$$\frac{\overline{EF}}{2}=\frac{8}{5}$$

Udaljenost $\overline{CM}$ računa se kao:

$$\overline{CM}=\frac{|AX+BY+C|}{\sqrt{A^2+B^2}}=\frac{|3x_C+4y_C-1|}{\sqrt{9+16}}=\frac{|3\cdot1+4\cdot1-1|}{5}=\frac{6}{5}$$

$$\overline{EC}=\overline{FC}=r=\sqrt{\overline{CM}^2+\overline{FM}^2}=\sqrt{(\frac{6}{5})^2+(\frac{8}{5})^2}=2$$

Jednačina kružnice:

k: $(x-1)^2+(y-1)^2=4$

$$\overline{AC}=\sqrt{(x_A-x_C)^2+(y_A-y_C)^2}=\sqrt{(1-1)^2+(1-5)^2}=4$$

$$\sin\frac{\varphi}{2}=\frac{r}{\overline{AC}}=\frac{2}{4}=\frac{1}{2}$$

$$\frac{\varphi}{2}=30^o \qquad (\frac{\varphi}{2}=\frac{\pi}{6})$$

$$\frac{\varphi}{2}=60^o \qquad (\frac{\varphi}{2}=\frac{\pi}{3})$$